Techniques for Computing Rate and Volume of Stream Depletion by Wells[a]

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ABSTRACT

The effects on flow of a nearby stream from pumping a well can be calculated readily using dimensionless curves and tables. Computations can be made of: (1) the rate of stream depletion at any time during the pumping period or after the cessation of pumping; (2) the volume induced from the stream during any time, both during pumping or after the cessation of pumping; and (3) the effects, both in rate and volume of stream depletion, of any selected pattern of intermittent pumping. Sample computations illustrate the use of the curves and tables. An example shows that intermittent pumping may have a pattern of stream depletion not greatly different from a pattern for steady pumping of an equal volume.

The residual effects of pumping, that is, effects after cessation of pumping, on streamflow may easily be greater than the effects during the pumping period. Adequate advance planning that includes consideration of residual effects thus is essential to effective administration of a stream-aquifer system.

INTRODUCTION

With increasing frequency, problems of water administration require evaluation of effects of ground-water withdrawal on surface supplies. Both rate and volume effects have significance. Effects after the cessation of pumping (called residual effects in this paper) are important also and have not previously been examined in detail. In fact, residual effects can be much greater than those during pumping. Curves and tables shown in this paper, although applicable to a large range of interactions, are especially oriented to the solution of problems involving very small interactions, and to the evaluation of residual effects. Where many wells are concentrated near a stream, the combined withdrawals have a significant effect on the availability of water in the stream.

The relations between the pumping of a well and the resulting depletion of a nearby stream have been derived by several investigators (Theis, 1941; Conover, 1954; Glover and Balmer, 1954; Glover, 1960; Theis and Conover, 1963; and Hantush, 1964, 1965). The results generally are shown in the form of equations and charts. Except for the charts shown by Glover (1960) in a publication that had limited distribution, the charts are useful as computational tools only in the range of comparatively large effects, leaving the user with rather formidable equations to solve in evaluating small effects. The average user retreats in dismay when faced by the mysticism of "line source integral," "complementary error function," or "the second repeated integral of the error function." Because this writer definitely is a member of the community of "average users," he has exercised what he believes to be his prerogative of reversing the usual order of presentation. In this paper, the working tools – curves, tables, and sample computations – are shown first, and the discussion of their mathematical bases is relegated to an appendix. The usefulness of the tools will not be greatly enhanced by an understanding of the material in the appendix; it is shown for the benefit of those who desire to examine the mathematical bases of the tools.

The computations shown in this paper are not new, but they seem to have been rather well concealed from most users. Their value to water administrators is apparent, especially in the estimation of total volume of depletion and of residual effects. The primary purpose of this report is to provide tools and examples of their use that will simplify the seemingly intricate computations.

DEFINITIONS AND ASSUMPTIONS

The units used are defined below, except those having to do only with the mathematics, which are defined in the appendix:

T = the coefficient of transmissibility, $[L^2/T]$;

S = the specific yield of the aquifer, dimensionless;

t = time, during the pumping period, since pumping began, $[T]$;

t_p = total time of pumping, $[T]$;

t_i = time after cessation of pumping, $[T]$;

Q = the steady pumping rate, $[L^3/T]$;

q = the rate of depletion of the stream, $[L^3/T]$;

Qt = the total volume pumped during time t, $[L^3]$;

Qt_p = the total volume pumped, $[L^3]$;

v = the volume of stream depletion during time t, t_p, or $t_p + t_i$, $[L^3]$;

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a = the distance from the pumped well to the stream, [L];

sdf = the stream depletion factor, [T]. This term has not been used previously.

If the aquifer meets the assumptions listed in this section, the sdf is a^2S/T, which is the time required to make the term tT/a^2S unity. In any aquifer, however complex, it is the time from the beginning of steady pumping within which the volume of stream depletion is 28 percent of the volume pumped.

To avoid confusion due to the use of the same symbol for the dimension time as for the coefficient of transmissibility, symbols for the dimensions time and length are in Roman capitals and enclosed in brackets. All other symbols, except that designating the mathematical term, second repeated integral, are in italics.

The dimensions of T, t, Q, q, v, and a used in the sample computations are specified where necessary.

Stream depletion means either direct depletion from the stream or reduction of return flow to the stream.

The curves and tables in this report are dimensionless, and can be used with any dimensional system. The units in the system must be consistent, however. For example, if Q and q are in acre-ft/day (acre-feet per day), v must be in acre-ft (acre-feet). If a is in ft (feet), and T/S is in gal/day-ft (gallons per day per foot), the value of T/S must be converted to ft²/day (square feet per day). A T/S value of 10^6 gal/day-ft equals (10^6 gal/day-ft) ($\frac{1 \text{ ft}^3}{7.48 \text{ gal}}$) equals 133,700 ft²/day.

The assumptions made are the same as other investigators have made. They are as follows:

For a water-table aquifer, drawdown is considered to be negligible when compared to the saturated thickness; that is, T does not change with pumping time.

The aquifer is isotropic, homogeneous, and semi-infinite in areal extent, with a straight, fully penetrating stream boundary.

Water is released instantaneously from storage.

The well is fully penetrating.

The pumping rate is steady during times t and t_p.

The residual effects of previous pumping are negligible.

Studies underway by the writer and associates suggest that errors due to fairly large departures from idealized conditions can be reduced to tolerable proportions by using a model to determine the stream depletion factor. Results from an electric-analog model of a stream-aquifer system are being analyzed to determine the effects of complex boundary conditions and areal variations in aquifer properties on stream depletion factors.

DESCRIPTION OF CURVES AND TABLES

Effects During Pumping

Curves A and B in Figure 1 apply during the period of steady pumping. Curve A shows the relation between the dimensionless term tT/a^2S and the rate of stream depletion, q, at time t, expressed as a ratio to the pumping rate Q. Curve B shows the relation between tT/a^2S and the volume of stream depletion, v, during time t, expressed as a ratio to the volume pumped, Qt. The coordinates of both curves are tabulated in Table 1. The number of significant figures shown for the values tabulated in Table 1 was determined by needs for some of the computations described in the next section, which sometimes involve small differences between relatively large numbers. Precision to more than two significant figures in reporting results probably will never be warranted.

Residual Effects

If there is no capture of water from other sources, the stream will continue to lose water after pumping stops. As time approaches infinity, the volume of stream depletion approaches the volume pumped. The rate and volume of depletion at any time after cessation of pumping can be computed using the method of superposition, that is, by assuming that the pumping well continues to pump, and that an imaginary well at the same location is recharged continuously at the same rate the pumping well is discharging. Residual effects are shown in Figures 2 and 3 for five selected values of t_pT/a^2S. The t_i is the time after cessation of pumping.

Problems concerned with values of t_pT/a^2S other than those for which curves are shown in Figures 2 and 3 generally can be solved with an acceptable degree of accuracy by interpolation, but if the user desires a more accurate appraisal, separate computations can be made.

Because computations of residual effects sometimes involve small differences between large numbers, use of tabular values may be preferable. The computations shown in Table 2, which are the basis for the curves labeled $t_pT/a^2S = 0.35$ in Figures 2 and 3 and the curve in Figure 4, will serve as an illustration of how additional curves can be constructed.

The curve labeled q in Figure 4 shows the relation between q and t resulting from pumping a well 3,656 feet from a stream at a rate of 10 acre-ft/day for 35 days. The ratio of T/S is 13.37×10^4 ft²/day, which is not an unusual value for an alluvial aquifer. The sdf is 100 days. The pumping rate is 10 acre-ft/day; the maximum rate of stream depletion is 2.7 acre-ft/day. Pumping ceases at the end of 35 days; the maximum rate of stream depletion occurs about 10 days later, and q still is about half the maximum rate 45 days after cessation of pumping.

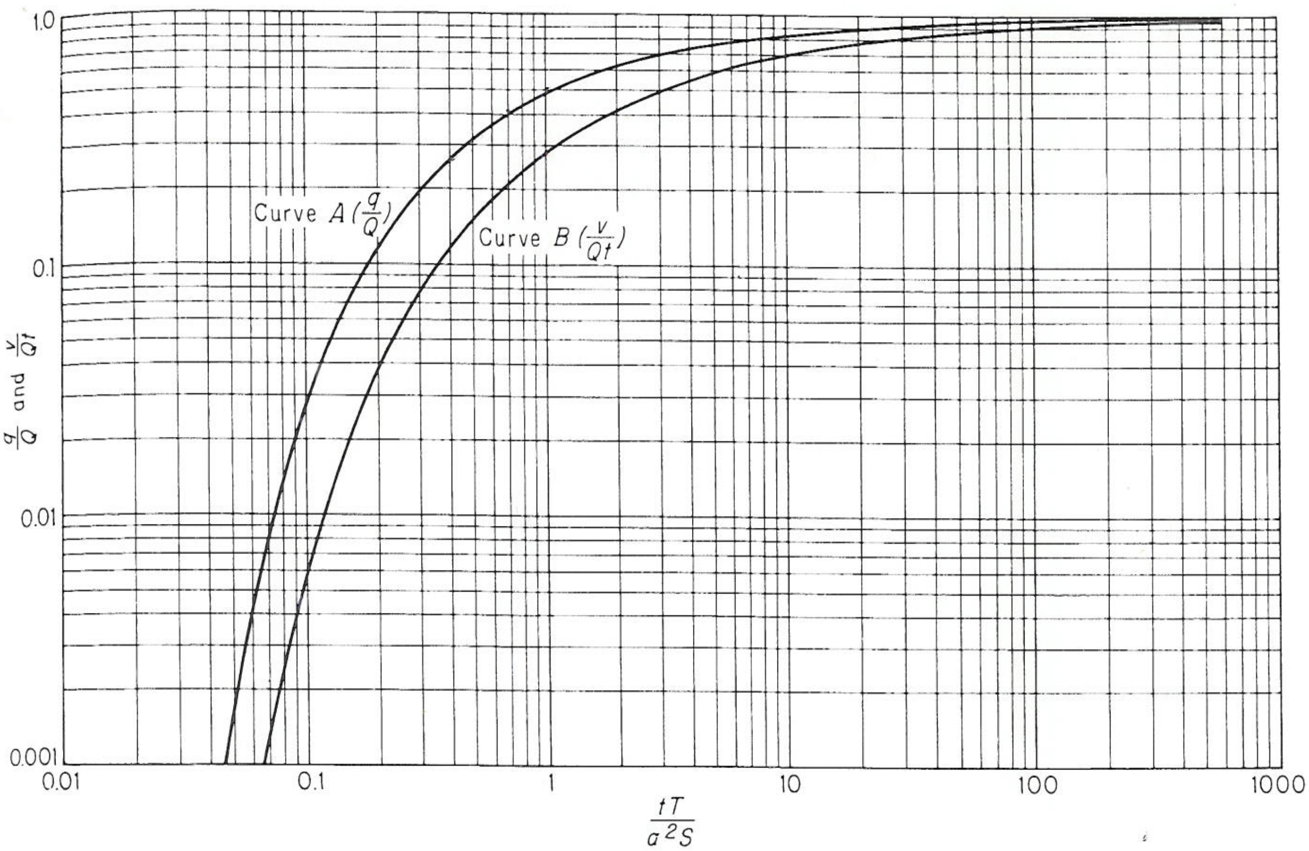

Fig. 1. Curves to determine rate and volume of stream depletion.

TABLE 1. VALUES OF q/Q, v/Qt, AND vT/Qa^2S CORRESPONDING TO SELECTED VALUES OF tT/a^2S

[tT/a^2S = time, in units of the *sdf*, dimensionless; q/Q = ratio of rate of depletion of the stream to the steady pumping rate, dimensionless; v/Qt = ratio of volume of stream depletion to volume pumped, dimensionless; vT/Qa^2S = volume of stream depletion in units of $(Q)(sdf)$, dimensionless.]

tT/a^2S	q/Q	v/Qt	vT/Qa^2S	tT/a^2S	q/Q	v/Qt	vT/Qa^2S
0	0	0	0	1.6	0.576	0.375	0.600
.07	.008	.001	.0001	1.7	.588	.387	.658
.10	.025	.006	.0006	1.8	.598	.398	.716
.15	.068	.019	.003	1.9	.608	.409	.777
.20	.114	.037	.007	2.0	.617	.419	.838
.25	.157	.057	.014	2.2	.634	.438	.964
.30	.197	.077	.023	2.4	.648	.455	1.09
.35	.232	.097	.034	2.6	.661	.470	1.22
.40	.264	.115	.046	2.8	.673	.484	1.36
.45	.292	.134	.060	3.0	.683	.497	1.49
.50	.317	.151	.076	3.5	.705	.525	1.84
.55	.340	.167	.092	4.0	.724	.549	2.20
.60	.361	.182	.109	4.5	.739	.569	2.56
.65	.380	.197	.128	5.0	.752	.587	2.94
.70	.398	.211	.148	5.5	.763	.603	3.32
.75	.414	.224	.168	6.0	.773	.616	3.70
.80	.429	.236	.189	7	.789	.640	4.48
.85	.443	.248	.211	8	.803	.659	5.27
.90	.456	.259	.233	9	.814	.676	6.08
.95	.468	.270	.256	10	.823	.690	6.90
1.0	.480	.280	.280	15	.855	.740	11.1
1.1	.500	.299	.329	20	.874	.772	15.4
1.2	.519	.316	.379	30	.897	.810	24.3
1.3	.535	.333	.433	50	.920	.850	42.5
1.4	.550	.348	.487	100	.944	.892	89.2
1.5	.564	.362	.543	600	.977	.955	573

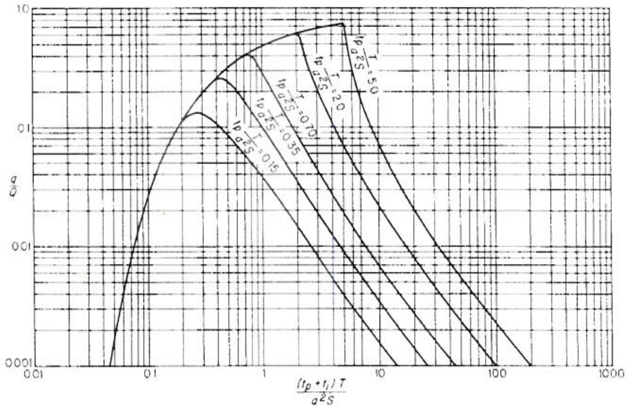

Fig. 2. Curves to determine rate of stream depletion during and after pumping.

The area in the rectangle labeled Q represents total volume pumped; the area under the curve labeled q represents the volume of stream depletion. In terms of volume removed from the stream during the pumping period, the effect is small, only about 10 percent of the volume pumped. However, the effect continues, and as time approaches infinity, the volume of stream depletion approaches the volume pumped.

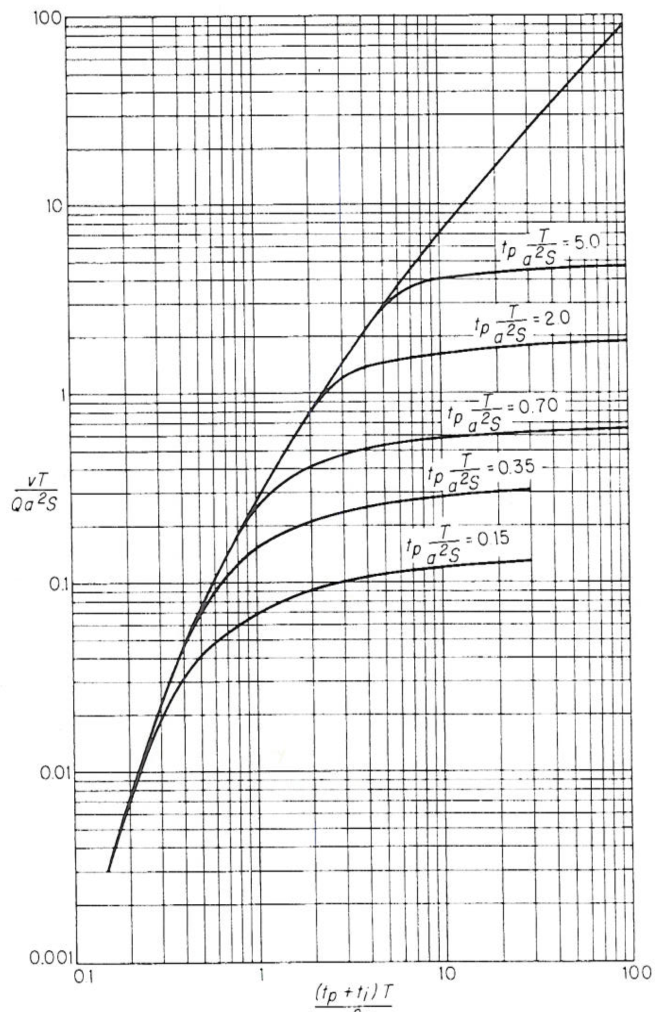

Fig. 3. Curves to determine volume of stream depletion during and after pumping.

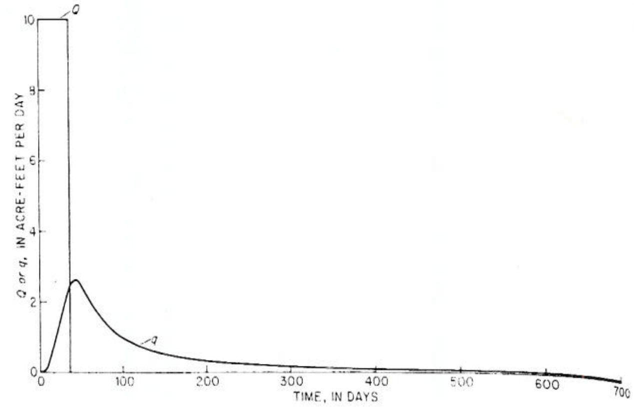

Fig. 4. Example of residual effects of well pumping 35 days.

Consideration of residual effects such as are illustrated in Figure 4 leads to the conclusion that effective administration of a system that uses both surface water and a connected ground-water reservoir requires a great deal of foresight. The immediate effects on streamflow of a change in pumping pattern may be very small; plans adequate for effective administration of the resource generally require consideration of needs in the future – sometimes the distant future. The sample problems solved later in this report illustrate the need for long-range plans in water administration.

The curves in Figure 5 illustrate the effect of one pattern of intermittent pumping. The computations are shown in Table 3. Effects on the stream, both in volume removed and rate of removal, are compared for two patterns of pumping of 48 acre-ft during a 32-day period. In both cases the aquifer has a ratio T/S of 13.37×10^4 ft^2/day and the well is 1,890 feet from the stream, making the *sdf* = 26.7 days. In the first case the well is pumped at a rate of 4 acre-ft/day for 4 days, shut down 10 days, pumped 4 days, shut down 10 days, and pumped 4 days. In the second case, the well is pumped continuously at a rate of 1.5 acre-ft/

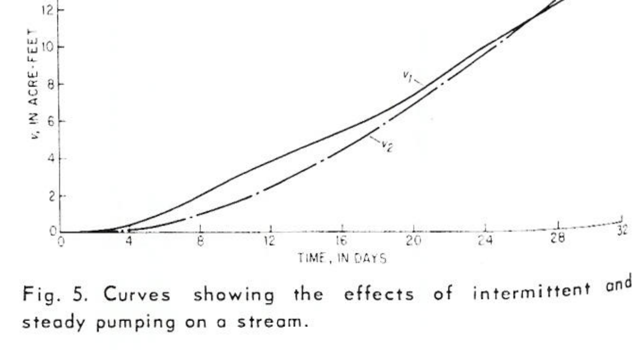

Fig. 5. Curves showing the effects of intermittent and steady pumping on a stream.

TABLE 2. COMPUTATION OF RESIDUAL EFFECTS OF PUMPING

[Pumping stopped when $tT/a^2S = 0.35$. See the explanation for Table 1 for definition of units.]

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0.35	0.232	0.034	0	0	0	0.232	0.034
.42	.275	.052	.07	.008	.0001	.267	.052
.45	.292	.060	.10	.025	.0006	.267	.059
.50	.317	.076	.15	.068	.003	.249	.073
.60	.361	.109	.25	.157	.014	.205	.095
.70	.398	.148	.35	.232	.034	.166	.114
1.00	.480	.280	.65	.380	.128	.099	.152
1.50	.564	.543	1.15	.510	.354	.053	.189
2.00	.617	.838	1.65	.581	.629	.035	.209
3.00	.683	1.49	2.65	.664	1.255	.019	.235
5.00	.752	2.94	4.65	.743	2.67	.009	.27
7.00	.789	4.48	6.65	.783	4.21	.006	.27
10.00	.823	6.90	9.65	.8198	6.61	.0032	.29
15.00	.855	11.1	14.65	.8528	10.81	.0022	.29
20.00	.872	15.4	19.65	.8718	15.00	.0012	.30
30.00	.897	24.3	29.65	.8961	23.99	.0009	.31

(1) $[(t_p + t_i)/a^2 S]\ T = tT/a^2 S$ for pumped well if pumping had continued.

(2) q/Q for pumped well if pumping had continued. Values from Table 1 for value of tT/a^2S indicated in Col. 1.

(3) vT/Qa^2S for pumped well if pumping had continued. Values from Table 1 for value of tT/a^2S indicated in Col. 1.

(4) tT/a^2S for recharged well, beginning at cessation of pumping.

(5) q/Q for recharged well, beginning at cessation of pumping. Values from Table 1 for value of tT/a^2S indicated in Col. 4.

(6) vT/Qa^2S for recharged well, beginning at cessation of pumping. Values from Table 1 for value of tT/a^2S indicated in Col. 4.

(7) Col. 2 minus Col. 5; residual q/Q.

(8) Col. 3 minus Col. 6; residual vT/Qa^2S.

day for 32 days. The computed effects of the pattern of intermittent pumping are compared to those of the steady rate in Figure 5. The comparisons indicate that, within quite large ranges of intermittency, the effects of intermittent pumping are approximately the same as those of steady, continuous pumping of the same volume.

SAMPLE COMPUTATIONS

To illustrate the use of the curves and tables, solutions are shown of problems that might arise in the conjunctive administration of ground water and surface water.

Problem I

If administrative criteria require that pumping cease when the rate of stream depletion by pumping reaches 0.14 acre-ft/day: (1) Under this restriction how long can a well ½ mile from the stream be pumped at the rate of 2 acre-ft/day if T/S is 10^5 gal/day-ft, and what is the volume of stream depletion during this time? (2) If pumping this well is stopped when $q = 0.14$

TABLE 3. COMPUTATION OF THE EFFECTS OF A SELECTED PATTERN OF INTERMITTENT PUMPING ON A NEARBY STREAM

[See Tables 1 and 2 for definition of units: $a = 1{,}890$ ft, $T/S = 13.37 \times 10^4$ ft² per day, $sdf = 26.7$ days. Pump on 4 days, off 10 days, on 4 days, off 10 days, and on 4 days; $Q_1 = 4$ acre-ft per day, $t_{p_1} T/a^2S = 0.15$ (see curves in Figures 3 and 4) for each pumping period. $Q_2 = 1.5$ acre-ft per day, $t_{p_2} = 32$ days.]

Pumping period (1st-4th day inclusive)				Pumping period (15th-18th day inclusive)				Pumping period (29th-32nd day inclusive)				Totals				$t_{p_2} = 32$ days, $Q_2 = 1.5$ acre-ft per day			
Time (days)	tT/a^2S	q/Q	vT/Qa^2S	Time (days)	tT/a^2S	q/Q	vT/Qa^2S	Time (days)	tT/a^2S	q/Q	vT/Qa^2S	q/Q	q (acre-ft/day)	vT/Qa^2S	v (acre-ft)	q/Q	q (acre-ft/day)	vT/Qa^2S	v (acre-ft)
0	0	0	0	–	–	–	–	–	–	–	–	0	0	0	0	0	0	0	0
4	.15	.068	.003	–	–	–	–	–	–	–	–	.068	.27	.003	.3	.068	.10	.003	.12
7	.26	.132	.014	–	–	–	–	–	–	–	–	.132	.53	.014	1.5	.165	.25	.016	.64
14	.52	.080	.043	0	0	0	0	–	–	–	–	.080	.32	.043	4.6	.326	.49	.081	3.2
18	.67	.060	.055	4	.15	.068	.003	–	–	–	–	.128	.51	.058	6.2	.388	.58	.135	5.4
21	.79	.050	.060	7	.26	.132	.014	–	–	–	–	.182	.73	.074	7.9	.426	.64	.185	7.4
28	1.05	.034	.070	14	.52	.080	.043	0	0	0	0	.114	.46	.113	12.1	.490	.74	.305	12.2
32	1.20	.029	.075	18	.67	.060	.055	4	.15	.068	.003	.157	.63	.133	14.2	.519	.78	.38	15.2

acre-ft/day, what will the rate of stream depletion be 30 days later? What will be the volume of stream depletion at that time? (3) What will be the largest rate of stream depletion and when will it occur?

$$sdf = a^2S/T = \frac{a^2}{T/S} = \frac{(\tfrac{1}{2}\text{ mi})^2(5280\text{ ft/mi})^2}{(10^5\text{ gal/day-ft})(1\text{ ft}^3/7.48\text{ gal})} = 520\text{ days.}$$

Part (1)

$$q/Q = \frac{0.14\text{ acre-ft/day}}{2\text{ acre-ft/day}} = 0.07$$

From Curve *A* (Figure 1):

$tT/a^2S = 0.15$

$t = (0.15)(sdf) = (0.15)(520\text{ days}) = 78\text{ days}$

From Curve *B* (Figure 1): if

$tT/a^2S = 0.15$,

$v/Qt = 0.02$

$v = (0.02)(2\text{ acre-ft/day})(78\text{ days}) = 3.1$ acre-ft.

During the 78-day pumping period, 3.1 acre-ft, out of a total of 156 acre-ft pumped, is stream depletion.

Part (2)

If pumping is stopped at the end of 78 days, then

$t_pT/a^2S = 0.15$,

and 30 days later,

$$(t_p + t_i)T/a^2S = \frac{108\text{ days}}{520\text{ days}} = 0.21.$$

From Figure 2: if

$t_pT/a^2S = 0.15$

and

$(t_p + t_i)T/a^2S = 0.21$,

$q/Q = 0.12$,

$q = (0.12)(2\text{ acre-ft/day}) = 0.24$ acre-ft/day, 30 days after pumping stops.

From Figure 3:

$vT/Qa^2S = 0.008$,

$v = (0.008)(2\text{ acre-ft/day})(520\text{ days}) = 8.3$ acre-ft of stream depletion during 108 days as a result of pumping 2 acre-ft/day during the first 78 days.

Part (3)

From Figure 2: if

$t_pT/a^2S = 0.15$,

maximum $q/Q = 0.13$

when

$(t_p + t_i)T/a^2S = 0.25$,

therefore, maximum $q = 0.13$ (2 acre-ft/day) $= 0.26$ acre-ft/day when

$t_p + t_i = (0.25)(520\text{ days}) = 130$ days, or 52 days after pumping stops.

Problem II

An irrigator is restricted to a maximum withdrawal of 150 acre-ft during the 150-day growing season, provided his pumping depletes the stream less than 25 acre-ft during the season. His well is 1 mile from the stream and $T/S = 133{,}700\text{ ft}^2$/day. Examine the effects of several possible pumping patterns:

$$sdf = a^2S/T = \frac{a^2}{T/S} = \frac{(5280\text{ ft})^2}{(133700\text{ ft}^2/\text{day})} = 209\text{ days.}$$

Part (1)

First, test to see if both restrictions apply to any combination of pumping time and rate within the 150-day period. Try ending pumping the last day of the season, beginning pumping at a time and rate such that pumping 150 acre-ft will result in a depletion of the stream of 25 acre-ft at the end of pumping.

$Qt = 150$ acre-ft, $v = 25$ acre-ft, $v/Qt = 0.167$.

From Curve *B*:

$tT/a^2S = 0.54$,

$t = 0.54\ (sdf)$

$= 0.54\ (209\text{ days}) = 113$ days, or 37 days after beginning of season.

$$Q = \frac{150\text{ acre-ft}}{113\text{ days}} = 1.33\text{ acre-ft/day.}$$

Since both restrictions apply to a 113-day pumping period that ends at the end of the season, intuitive reasoning suggests that the restriction on volume pumped will dictate the pumping rate during shorter periods at the end of the season, and that the restriction on volume of stream depletion will dictate both rate and time of pumping for periods in the first part of the season.

Part (2)

Begin pumping 60 days after the beginning of the season. Test reasoning that the restriction on volume pumped applies.

$Qt = 150$ acre-ft,

$t = 90$ days,

$$tT/a^2S = \frac{90\text{ days}}{209\text{ days}} = 0.43.$$

From Curve *B*:

$v/Qt = 0.13$

$v = 0.13\ (150\text{ acre-ft}) = 19.5$ acre-ft,

therefore, the restriction on volume pumped does apply.

$$\text{pumping rate} = \frac{150 \text{ acre-ft}}{90 \text{ days}} = 1.67 \text{ acre-ft/day}.$$

Part (3)

Begin pumping at the beginning of the season, pump for 73 days. Test reasoning that the restriction on stream depletion applies.

$$t_pT/a^2S = \frac{73 \text{ days}}{209 \text{ days}} = 0.35$$

From Figure 3: for

$$tT/a^2S = 0.35$$

and

$$(t_p+t_i)T/a^2S = \frac{150 \text{ days}}{209 \text{ days}} = 0.72,$$

$$vT/Qa^2S = 0.12,$$

$$Q = \frac{25 \text{ acre-ft}}{(0.12)\ (209 \text{ days})} = 1.00 \text{ acre-ft/day},$$

$$Qt = (1.00 \text{ acre-ft/day})(73 \text{ days}) = 73 \text{ acre-ft}.$$

Therefore, the restriction on volume of stream depletion does apply, and dictates a pumping rate of not more than 1.00 acre-ft/day for a 73-day pumping period at the beginning of the season.

Part (4)

The irrigator elects to pump 0.5 acre-ft/day for the first 32 days of the season. What is the highest rate he can pump during the remaining 118 days?

Try assumption that restriction on volume of stream depletion will apply.

$$t_pT/a^2S = \frac{32 \text{ days}}{209 \text{ days}} = 0.15,$$

$$(t_p+t_i)T/a^2S = \frac{150 \text{ days}}{209 \text{ days}} = 0.72.$$

From Figure 3:

$$v_1T/Qa^2S = 0.057$$

$$v_1 = (0.057)\ (0.5 \text{ acre-ft/day})\ (209 \text{ days})$$

$$= 6.0 \text{ acre-ft}$$

$$Q_1t_1 = (0.5 \text{ acre-ft/day})\ (32 \text{ days}) = 16 \text{ acre-ft}$$

$$25 \text{ acre-ft} - 6 \text{ acre-ft} = 19 \text{ acre-ft} = v_2$$

$$t_2 = 118 \text{ days}$$

$$t_2T/a^2S = \frac{118 \text{ days}}{209 \text{ days}} = 0.56.$$

From Figure 1:

$$v_2/Q_2t_2 = 0.17$$

$$Q_2t_2 = 19 \text{ acre-ft}/0.17 = 112 \text{ acre-ft}$$

(112 + 16) acre-ft = 128 acre-ft, which is less than 150 acre-ft,

therefore the assumption that volume of stream depletion applies is correct.

$$Q_2 = \frac{112 \text{ acre-ft}}{118 \text{ days}} = 0.95 \text{ acre-ft/day}.$$

The irrigator elects to pump at the rate of 2.00 acre-ft/day. (5) If he plans to pump until the end of the season, how soon can he start pumping? (6) If he plans to start pumping at the beginning of the season, how long can he pump? (7) If he plans to start pumping 50 days after the beginning of the season, how long can he pump?

Part (5)

$$Qt = 150 \text{ acre-ft}$$

$$t = 150 \text{ acre-ft}/2 \text{ acre-ft/day} = 75 \text{ days}$$

$$tT/a^2S = 75 \text{ days}/209 \text{ days} = 0.36.$$

From Curve *B*:

$$v/Qt = 0.10$$

$$v = 15.0 \text{ acre-ft}$$

therefore the restriction on volume pumped applies.

Part (6)

Assume that the restriction on stream depletion applies,

$$vT/Qa^2S = \frac{25 \text{ acre-ft}}{2 \text{ acre-ft/day}\ (209 \text{ days})} = 0.060$$

$$(t_p+t_i)T/a^2S = \frac{150 \text{ days}}{209 \text{ days}} = 0.72.$$

From Figure 3:

$$t_pT/a^2S \cong 0.17$$

$$t_p = (0.17)\ (209) \cong 35\tfrac{1}{2} \text{ days}$$

check by using Table 1

$(t_p+t_i)T/a^2S$	v_1T/a^2S	t_iT/a^2S	v_iT/Qa^2S	Net vT/Qa^2S	Net Acre-ft
0.72	0.156	0.55	0.092	0.064	27
try $t_p = 33\tfrac{1}{2}$ days, $t_pT/a^2S = 0.16$					
0.72	0.156	0.56	0.095	0.061	25

therefore the irrigator can begin pumping at the beginning of the season and pump at a rate of 2 acre-ft/day for 33½ days.

Part (7)

Restriction on volume pumped limits pumping time to: 150 acre-ft/2 acre-ft/day = 75 days.

Test to see if depletion restriction would be exceeded by 75 days of pumping

$$t_p + t_i = (150 - 50) \text{ days} = 100 \text{ days}$$

$$(t_p+t_i)T/a^2S = \frac{100 \text{ days}}{209 \text{ days}} = 0.48$$

$$t_p = 75 \text{ days}$$

$$t_pT/a^2S = \frac{75 \text{ days}}{209 \text{ days}} = 0.36$$

From Figure 3:

$$vT/Qa^2S \cong 0.72$$

$$v \cong (0.72)(2\text{ acre-ft/day})(209\text{ days})$$

$$= 30 \text{ acre-ft}$$

which exceeds the 25 acre-ft restriction.

Try stopping pumping after 69 days.

Using Table 1:

$$t_i = (100 - 69)\text{ days} = 31\text{ days}$$

$(t_p+t_i)T/a^2S$	v_1T/Qa^2S	t_iT/a^2S	v_2T/Qa^2S	*Net*	*Acre-ft*
0.48	0.070	0.15	0.003	0.067	28

try t_p = 54 days, t_i = 46 days

0.48	0.070	0.22	0.010	0.060	25

Therefore, the irrigator can pump at a rate of 2 acre-ft/day during the 54-day period beginning 50 days after the season begins.

Problem III

A well 4,000 feet from the stream is shut down after pumping at the rate of 250 gal/min for 150 days: T/S = 66,850 ft²/day. (1) What effect did pumping the well have on the stream during the pumping period? (2) What will be the effect during the next 216 days after pumping was stopped? (3) What would the effect have been if pumping had continued during the entire 366 days?

$$sdf = \frac{(4000\text{ ft})^2}{66850\text{ ft}^2/\text{day}} = 239\text{ days.}$$

Part (1)

$$t_pT/a^2S = \frac{150\text{ days}}{239\text{ days}} = 0.63$$

$$Q = (250\text{ gal/min})(1440\text{ min/day})\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{1\text{ acre-ft}}{43560\text{ ft}^3}\right) = 1.1\text{ acre-ft/day}$$

when

$$t = t_p,\ tT/a^2S = 0.63,$$

and from Curve A:

$$q/Q = 0.37,$$

and from Curve B:

$$v/Qt = 0.19.$$

At the end of 150 days,

$$q = (1.1\text{ acre-ft/day})(0.37) = 0.41\text{ acre-ft/day}$$

$$v = (1.1\text{ acre-ft/day})(150\text{ days})(0.19) = 31\text{ acre-ft.}$$

Part (2)

When

$$t_p + t_i = (150 + 216)\text{ days} = 366\text{ days,}$$

$$(t_p + t_i)T/a^2S = 1.53.$$

From Figure 2: by interpolation,

$$q/Q \cong 0.10.$$

From Figure 3: by interpolation,

$$vT/Qa^2S \cong 0.32.$$

Compare results using Table 1 and method of superposition (see Table 2) with the estimates by interpolation.

$(t_p + t_i)T/a^2S$	q_p/Q	v_pT/Qa^2S	t_iT/a^2S	q_i/Q
1.53	0.568	0.560	0.90	0.456

v_iT/Qa^2S	$(q_p - q_i)/Q$	$(v_p - v_i)T/Qa^2S$
0.233	0.11	0.33

The results by interpolation in Figures 2 and 3 are very close to those using Table 1.

216 days after pumping ceased, q/Q = 0.11 and $v = (0.33)(Qa^2S/T) = 0.33$ (1.1 acre-ft/day) (239 days) = 87 acre-ft.

The additional volume of stream depletion during the 216-day period = (87 − 31) acre-ft = 56 acre-ft.

Part (3)

If pumping had continued for the entire 366-day period, tT/a^2S = 1.53, and (see tabulation in (2) above), q/Q = 0.568, and v/Qt = 0.366.

$$q = (0.568)(1.1\text{ acre-ft/day}) = 0.62\text{ acre-ft/day}$$

$$v = (0.366)(1.1\text{ acre-ft/day})(366\text{ days})$$

$$= 147\text{ acre-ft}$$

v during last 216 days = (147 − 31) acre-ft = 116 acre-ft.

Problem IV

A municipal well is to be drilled in an alluvial aquifer near a stream. Downstream water uses require that depletion of the stream be limited to no more than 5,000 cubic meters during the dry season, which commonly is about 200 days long. The well will be pumped continuously at the rate of 0.03 m³/sec (cubic meters per second). T = 30 cm²/sec (square centimeters per second), and S = 0.20. What is the minimum allowable distance between the well and the stream?

$$Qt = (0.03\text{ m}^3/\text{sec})(200\text{ days})(86400\text{ sec/day}) = 5.184(10)^5\text{ m}^3$$

$$v/Qt = 5000\text{ m}^3/5.184(10)^5\text{ m}^3 = 0.01.$$

From Curve B:

$$tT/a^2S = 0.12$$

$$= \frac{(200\text{ days})(86400\text{ sec/day})(30\text{ cm}^2/\text{sec})}{a^2(0.20)}$$

$$a^2 = \frac{(200)(86400)(30)\text{cm}^2}{(0.12)(0.20)} = 2.16(10)^{10}\ \text{cm}^2$$

$$a = 1.47(10)^5\ \text{cm}$$

$$= 1470\ \text{meters.}$$

APPENDIX

The literature concerning the effect of a pumping ›ll on a nearby stream contains several equations d charts that, although superficially greatly different, eld identical results. The basic curves and table :urves A and B, and Table 1) of this report can be rived from any of the published expressions. A ırsory review of some of the pertinent equations may useful to those interested in the mathematics.

efinitions

The notation that has been used in the literature even more diverse than the published equations, nsequently, definitions of only selected terms are ven below. Complete definitions of all terms used e in the indicated references:

erf x = the error function of $x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = 1 - \text{erfc}\ x$

erfc x = the complementary error function of $x = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$

i^2 erfc x = the second repeated integral of the error function.

The line source integral (Maasland and Bittinger, 963, p. 84)

$$= \sqrt{\pi} \int_{x/\sqrt{4h^2 t}}^{\infty} \frac{e^{-u^2}}{u^2}\, du.$$

In the notation used in the main body of this re- ort,

$$x/\sqrt{4h^2 t} = \frac{a}{\sqrt{4tT/S}}.$$

Definitions and tabular values of erf x, erfc x, and i^2 erfc x are shown by Gautschi (1964, p. 297, 310-311, 316-317). Tabular values of the line source inte- ;ral are shown by Maasland and Bittinger (1963, p. 34), and by Glover (1964, pp. 45-53).

Aathematical Base for Curve A

Curve A and its coordinates in Table 1 can be :omputed from (Theis, 1941; Conover, 1954; Theis ınd Conover, 1963):

$$P = \frac{2}{\pi} \int_0^{\pi/2} e^{-k\, sec^2\, u}\, du \quad \ldots\ldots (1)$$

rom (Glover and Balmer, 1954):

$$q/Q = 1 - P\left(\frac{x_1}{\sqrt{4at}}\right) \quad \ldots\ldots\ldots\ldots (2)$$

from (Glover, 1960):

$$q_1/Q = 1 - \frac{2}{\sqrt{\pi}} \int_0^{x_1/\sqrt{4at}} e^{-u^2} du \quad \ldots (3)$$

and from (Hantush, 1964, 1965):

$$Q_r = Q\ \text{erfc}\,(U) \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Theis transformed his basic integral into equation (1) because the basic integral is laborious to evaluate, but in the form of equation (1), is amenable to either numerical or graphical solution. Equations (2), (3), and (4) are identical, and in the notation used in this paper are:

$$q/Q = \text{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) = 1 - \text{erf}\left(\frac{a}{\sqrt{4tT/S}}\right) \quad \ldots (5)$$

Mathematical Base for Curve B

Curve B and its coordinates in Table 1 can be computed either by integration of Curve A or of the equations that are the base of Curve A. Analytical integration of equations (2) and (3) is shown by Glover (1960) as:

$$\int_0^t \frac{q_r\, dt}{Qt} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{x_1/\sqrt{4at}} e^{-u^2} du - \frac{2}{\pi}\left(\frac{x_1^2}{4at}\right)\sqrt{\pi} \int_{x_1/\sqrt{4at}}^{\infty} \frac{e^{-u^2}}{u^2}\, du \quad \ldots (6)$$

and by (Hantush, 1964, 1965):

$$v_r = \int_0^{t_o} Q_r\, dt = 4Qt_o\ i^2\,\text{erfc}\,(U_o) \quad \ldots (7)$$

In the notation used in this paper, (6) is:

$$v/Qt = 1 - \text{erf}\left(\frac{a}{\sqrt{4tT/S}}\right) - \frac{2}{\pi}\left(\frac{a^2}{4tT/S}\right)\sqrt{\pi} \int_{a/\sqrt{4tT/S}}^{\infty} \frac{e^{-u^2}}{u^2}\, du \ldots (8)$$

and (7) is:

$$v/Qt = 4\ i^2\,\text{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) \quad \ldots\ldots\ldots (9)$$

Equations (8) and (9) both can be expressed in terms extensively tabulated in Gautschi (1964, pp. 310-311) as:

$$v/Qt = \left(\frac{a^2}{2tT/S} + 1\right) \text{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) - \left(\frac{a}{\sqrt{4tT/S}}\right)\left(\frac{2e^{-a^2/(4tT/S)}}{\sqrt{\pi}}\right) \quad \ldots (10)$$

Before discovering equations (6) and (7), the writer integrated Curve A both numerically and graphically. The results were identical, within the limitations of the methods, to those obtained from (10).

In the interest of simplicity and convenience, the term tT/a^2S was used as the independent variable of Curves A and B and Table 1, rather than the term $a/\sqrt{4tT/S}$, which has been used by other writers.

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